

HOMEWORK SET 7: 3-D SCHRÖDINGER EQUATION I

Due Wednesday, February 12, 2025

PROBLEMS FROM TZDII¹

1) **8.43** The probability of finding an electron in the region $r > a$ is $\int_a^\infty P(r) dr$. What is the prob-

ability that a 1s electron in hydrogen would be found outside the Bohr Radius, a_B ? (LOOK UP R_{1s} IN TABLE 8.2, FOLLOW INSTRUCTIONS IN THE FOOTNOTE OF THE TABLE TO WRITE THE PROBABILITY DENSITY, $P(r)$, THEN INTEGRATE BY PARTS. DOES YOUR ANSWER MAKE SENSE? MAKE A PHYSICAL ARGUMENT AS TO WHY IT DOES OR DOES NOT.)

2) **8.47 a)** Write down the θ equation (8.65) for the 2p states with $m = \pm 1$. Show that the solution is $\Theta(\theta) = \sin(\theta)$. This means that the complete wave functions for the 2p states with $m = \pm 1$ are

$$\psi_{2,1,\pm 1} = R_{2p}(r) \sin(\theta) e^{\pm i\phi}$$

b) Prove that the sum of these two wave functions is the $2p_x$ wave function (times an uninteresting factor of 2) and that the difference is the $2p_y$ function (times $2i$). [Hint: Rewrite $e^{\pm i\phi}$ as $\cos(\phi) \pm i\sin(\phi)$ and remember the relations for x and y in terms of r , θ , and ϕ in Fig. 8.11.]. Comment on what this means in a radial potential.

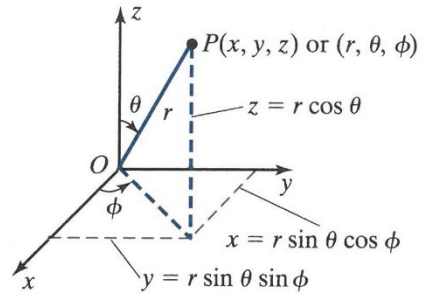
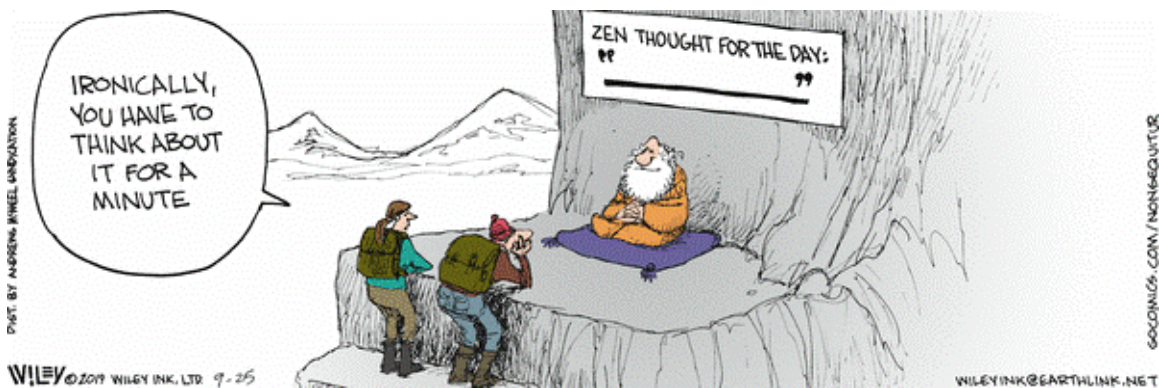


FIGURE 8.11

The spherical polar coordinates of a point P are (r, θ, ϕ) , where r is the distance OP , θ is the angle between OP and the z axis, and ϕ is the angle between the x - z plane and the vertical plane containing OP .



¹ Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Edition, Pearson, Prentice Hall, 2004